Popper on Verisimilitude

Two key notions difference of Truth content & falsity content.

It is a property of a false theory – how nearly is it true.

Prob. (theory is true) is a measure of how ~~sole~~ near to truth we think the theory is in the light of present evidence

But evaluation of a theory is time-dependent to this extent.

A true theory may not be known to be true (cf Xenophanes quotation).

So Two questions

(1) Ontologically How true is a theory

(2) what rationally should we do ~~to~~ believe about the truth of a theory.

A good theory is ~~to~~ one which we believe to be true for good reasons (not one which is true)

→ Prob (research Programme is true)

3 sorts of adhocness

adhoc 1 — no novel predictions

adhoc 2 — no confirmed novel predictions

adhoc 3 — not in accord. with the
heuristic of programme.

novelty prediction is novel if it does not be
in problem situation which gave
rise to the theory.

and $P(X_{n+1})$

$$= P(X_{n+1}/T)\,P_n + P(X_{n-1}/\sim T)(1-P_n)$$

$$= P_n + \varepsilon(1-P_n) = \varepsilon + P_n(1-\varepsilon)$$

In usual notation $P(F/X_1)$

$$\rightarrow P(\sim T/G)$$

and write $P(T/X_1 \& a)$

$$= \frac{P(X_1/T)\,P(T/a)}{P(X_1/T)\,P(T/a) + P(X_1/\sim T)\,P(\sim T/a)}$$

$$= \frac{x}{x + \varepsilon(1-x)}$$

$$= \frac{1}{1-\varepsilon + \varepsilon/x}$$

then $P_n = \cancel{\Omega(P_{n-1})}\ \Omega(P_{n-1}) \cdots \Omega(P_0)\cdot \cancel{x}\, P_0$

$$\left.\begin{aligned} P_{s+1} &= \Omega(P_s)\,P_s \\ \Omega(P_s) &= \frac{1}{P_s(1-\varepsilon)+\varepsilon} \end{aligned}\right\}$$

solve to give $P_n = \dfrac{1}{1-\varepsilon^n + \varepsilon^n/x}$

$$P(X_{n+1}) = \varepsilon + \frac{1-\varepsilon}{1-\varepsilon^n + \varepsilon^n/x}$$

analysis of adhocness

Consider a theory q which explains a "fact" X_1 Suppose we have in mind the collection of all theories $C(X_1)$ which explain X_1. Then $q \in C(X_1)$ and if we denote the "true" theory by t then $t \in C(X_1)$. clearly any member of $C(X_1) - \{t\}$ will be a "false" theory.

We denote probability that q is actually t by $P(T|X_1)$, then probability that q is not t but belongs to $C(X_1)$ is denoted by $P(F|X_1)$
$= 1 - P(T|X_1)$

Consider a new fact X_2.
Denote by $P(X_2|T)$ the prob. that X_2 is explained by t, i.e. if q is true then we get X_2.

clearly $P(X_2|T) = 1$

~~Denote~~ Denote by $P(T|X_2, X_1)$ prob. q is true, knowing it explains X_2

$P(X_2|F, X_1)$ prob. ~~q is~~ we find X_2 when q is false, and belong to set $C(X_1)$ (i.e. is "fitted" w.r.t. fact X_1)

then by Bayes theorem

$$P(T|X_2,X_1) = \frac{P(X_2|T)\,P(T|X_1)}{P(X_2|T)\,P(T|X_1) + P(X_2|F,X_1)\,P(F|X_1)}$$

$$= \frac{P(T|X_1)}{P(T|X_1) + P(X_2|F,X_1)\left(1 - P(T|X_1)\right)}$$

write $P(X_2|F,X_1) = \varepsilon$

$P(T/X_1) = x$

then $P(T/X_2,X_1) = P_T(X_2,X_1)$, & $x = P_T(X_1)$

$$y = P_T(X_2,X_1) = \frac{x}{x + \varepsilon(1-x)} = \frac{x}{x(1-\varepsilon)+\varepsilon}$$

$$= \frac{1}{1-\varepsilon + \varepsilon/x}$$

Define $r = r(X_2,X_1) = \dfrac{P_T(X_2,X_1)}{P_T(X_1)} = \dfrac{y}{x}$

$$= \frac{1}{x(1-\varepsilon)+\varepsilon}$$

In general write

$$\Omega(X_n, X_{n-1} \cancel{\dots X_1}) = \frac{P_T(X_n, X_{n-1} \dots X_1)}{P_T(X_{n-1} \dots X_1)}$$

then $P_T(X_n \dots X_1) = \Omega(X_n, X_{n-1}) \dots \Omega(X_3, X_2)\,\Omega(X_2, X_1)\,P_T(X_1)$

now $\Omega(X_s, X_{s-1})$ is a function of $P_T(X_{s-1} \dots X_1)$

So we write $\Omega(X_s, X_{s-1}) = \Omega(P_{s-1})$

where $P_{s-1} = P_T(X_{s-1} \dots X_1)$

then $P_T(X_n \dots X_1) = \Omega(P_{n-1}) \dots \Omega(P_2)\,\Omega(P_1)\,P_T(X_1)$

& writing $P_T(X_n \dots X_1) = P_T(X_n)$

$$P_T(X_n) = \Omega(P_{n-1}) \dots \Omega(P_1)\,P_T(X_1)$$

where $\Omega(P_s) = \dfrac{\cancel{P}\;1}{P_s(1-\varepsilon) + \varepsilon}$

$$P_{s+1} = \Omega(P_s) \times P_s$$

or in abbreviated notation

$$P_n = \Omega(P_{n-1})\cdot\Omega(P_{n-2})\cdots\Omega(P_1)\,P_1 .$$

$$\left.\begin{aligned} \Omega(P_s) &= \frac{1}{P_s(1-\varepsilon)+\varepsilon} \\ P_{s+1} &= \Omega(P_s)\times P_s . \end{aligned}\right\}$$

So we can generate P_n from P_1 by recursion.

But also should not we have

$$P_n = \frac{P_1}{P_1+\varepsilon^{n-1}(1-P_1)} = \frac{1}{1-\varepsilon^{n-1}+\frac{\varepsilon^{n-1}}{P_1}} .$$

$$\frac{P_n}{P_1} = \frac{1}{P_1(1-\varepsilon^{n-1})+\varepsilon^{n-1}} \rightarrow \frac{1}{\varepsilon^{n-1}} \quad \text{as } P_1 \simeq 0 \ll \varepsilon^{n-1} .$$

as $n\to\infty$ $P_n \to 1$

the following becomes $P(X_n \cdots X_1 \mid F, X_1) = \varepsilon^{n-1}$.

To check.

Take $\rho_2 = \dfrac{\rho_1}{\rho_1 + \varepsilon^{\cancel{2}}(1-\rho_1)}$, $\rho_3 = \dfrac{\rho_1}{\rho_1+\varepsilon^2(1-\rho_1)}$

$$= \frac{1}{1-\varepsilon^2+\varepsilon^2/\rho_1}$$

By recursion $\quad \Omega(\rho_1) = \dfrac{1}{\rho_1 + \varepsilon(1-\rho_1)}$

$$\rho_2 = \Omega(\rho_1)\,\rho_1 = \frac{\rho_1}{\rho_1 + \varepsilon(1-\rho_1)} = \frac{1}{1-\varepsilon+\varepsilon/\rho_1}$$

$$\Omega(\rho_2) = \frac{1}{\rho_2 + \varepsilon(1-\rho_2)}$$

$$\rho_3 = \Omega(\rho_2) \times \rho_2 = \frac{\rho_2}{\rho_2+\varepsilon(1-\rho_2)} = \frac{1}{1-\varepsilon+\varepsilon/\rho_2}$$

$$= \cancel{\frac{\rho_1/(\rho_1+\varepsilon(1-\rho_1))}{\rho_1}} \qquad \rho_4 = \frac{1}{1-\varepsilon+\varepsilon/\rho_3}$$

$$\rho_3 = \frac{1}{1-\varepsilon + \varepsilon(1-\varepsilon+\varepsilon/\rho_1)}$$

$$= \frac{1}{1-\varepsilon+\varepsilon-\varepsilon^2+\varepsilon^2/\rho_1} = \frac{1}{1-\varepsilon^2+\varepsilon^2/\rho_1}$$

$$\rho_4 = \frac{1}{1-\varepsilon+\varepsilon(1-\varepsilon^2+\varepsilon^2/\rho_1)} = \frac{1}{1-\varepsilon^3+\varepsilon^3/\rho_1} \quad \ldots$$

In general.

$$p_{n+1} = \frac{1}{1-\varepsilon+\varepsilon/p_n}$$

$$p_n = \frac{1}{(1-\varepsilon^{n-1})+\frac{\varepsilon^{n-1}}{p_1}}$$ is soln because.

$p_{n+1} =$

$$\frac{1}{1-\varepsilon^n+\varepsilon^n/p_1} = \frac{1}{1-\varepsilon+\varepsilon\left(1-\varepsilon^{n-1}+\frac{\varepsilon^{n-1}}{p_1}\right)}$$

$$= \frac{1}{\frac{1}{1-\varepsilon^n+\varepsilon^n/p_1}}$$

Suppose now we have predicted $x_2 \ldots x_3 \ldots x_n$
what is probability that we will predict x_{n+1}

$$P(x_{n+1}) = P(x_{n+1}/T)\, P_T(x_n) + P(x_{n+1}/F, x_n \ldots x_1)\,(1 - P_T(x_n))$$

$$= P_T(x_n) + \varepsilon\,(1 - P_T(x_n))$$

$$= P_T(x_n)(1-\varepsilon) + \varepsilon$$

and $$\frac{P(x_{n+1})}{\varepsilon} = \frac{P_T(x_n)}{\varepsilon} + 1 - P_T(x_n)$$

$$= \left(\cancel{\varepsilon}\,\frac{x}{x + \varepsilon^{n-1}(1-x)}(1-\varepsilon) + \varepsilon\right)\Big/\varepsilon$$

$$= 1 + \frac{x(1-\varepsilon)}{\varepsilon x + \varepsilon^n(1-x)} \simeq 1 + \frac{x}{\varepsilon x + \varepsilon^n} \simeq 1 + \frac{1}{\varepsilon(1 + \varepsilon^{n-1}/x)}$$

as $n \to \infty$ $$\frac{P(x_{n+1})}{\varepsilon} \to 1/\varepsilon$$

$$= \frac{\varepsilon^n(1-x) + x}{\varepsilon^n(1-x) + \varepsilon x} \simeq \frac{\varepsilon^n + x}{\varepsilon^n + \varepsilon x} = \frac{1 + x/\varepsilon^n}{1 + x/\varepsilon^{n-1}}$$

h eg. $$\frac{P(x_3)}{\varepsilon} = \frac{1 + x/\varepsilon^2}{1 + x/\varepsilon}$$

Take. $x = .01$ (1% chance theory is true)

$\varepsilon = .1$ (10% chance getting 1 dec. place right)

$$P(X_{n+1}) \simeq \varepsilon + \frac{1}{1+\varepsilon^{n-1}/x} \quad \left(= \varepsilon + \frac{1-\varepsilon}{1-\varepsilon^{n-1}+\varepsilon^{n-1}/x} \right)$$

If $\dfrac{\varepsilon^{n-1}}{x}$ is small. $P(X_{n+1}) \longrightarrow 1+\varepsilon$

If $\dfrac{\varepsilon^{n-1}}{x}$ is large. $P(X_{n+1}) \to \varepsilon$

Take $x = .01$, $\varepsilon = .1$ $\qquad \dfrac{\varepsilon}{x} = 10 \qquad \dfrac{\varepsilon^2}{x} = 1, \quad \dfrac{\varepsilon^3}{x} = .1$

Then $P(X_2) = \varepsilon + \dfrac{1}{1+1/x} = \varepsilon + \dfrac{x}{1+x} = \dfrac{\varepsilon+x}{1+x} \simeq \varepsilon + x$

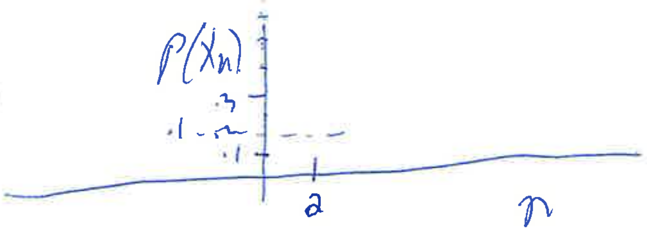
$= x(1-\varepsilon) + \varepsilon$

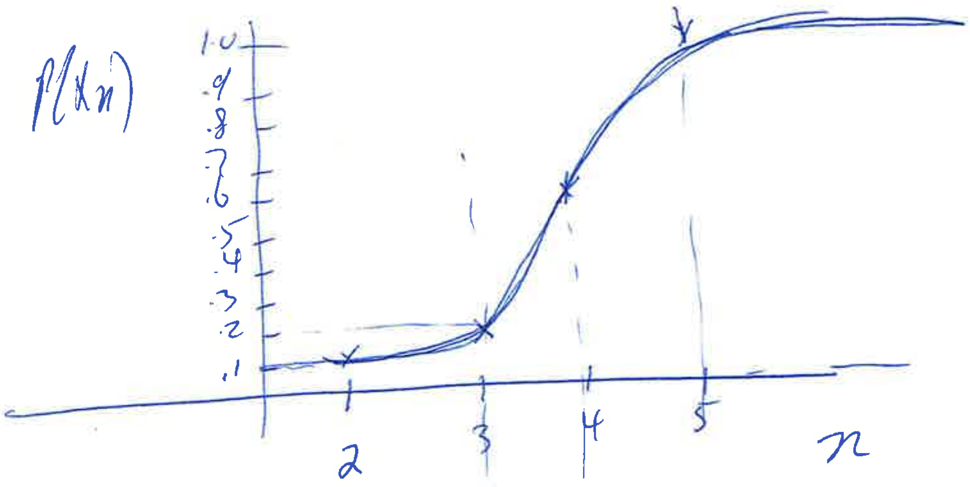
$= .1 + \dfrac{1}{1+100} \simeq .11$

$P(X_3) = .1 + \dfrac{1}{1+10} \simeq .2$

$P(X_4) = .1 + \dfrac{1}{1+1} = .1 + .5 = .6$

$P(X_5) = .1 + \dfrac{1}{1+.1} = .1 + \dfrac{1}{1.1} \simeq 1.00$





$\varepsilon^{n-1}/x = 1$ is where $P(X_n)$ changes very rapidly

Take n_0 a value of n for which $\frac{\varepsilon^{n_0-1}}{x} = 1$

then $n < n_0 - 1$ $P(X_n)$ is close to ε

$n > n_0 + 1$ $P(X_{n+1})$ is close 1

Now $P_T(X_n) = \frac{1}{1-\varepsilon^{n-1} + \varepsilon^{n-1}/x} = \frac{1}{1-\varepsilon^{n-1} + S}$

$S = \varepsilon^{n-1}/x$

S ~~small~~ large. $P_T(X_n)$ $P_T(X_n) \simeq 1/S = x \times \frac{1}{\varepsilon^{n-1}}$

S small $P_T(X_n) \simeq 1$

$S = 1$ $P_T(X_n) \simeq 1/2$.

If $x = 1/2$ then $P_T(X_n) = \frac{1}{1+\varepsilon^{n-1}}$

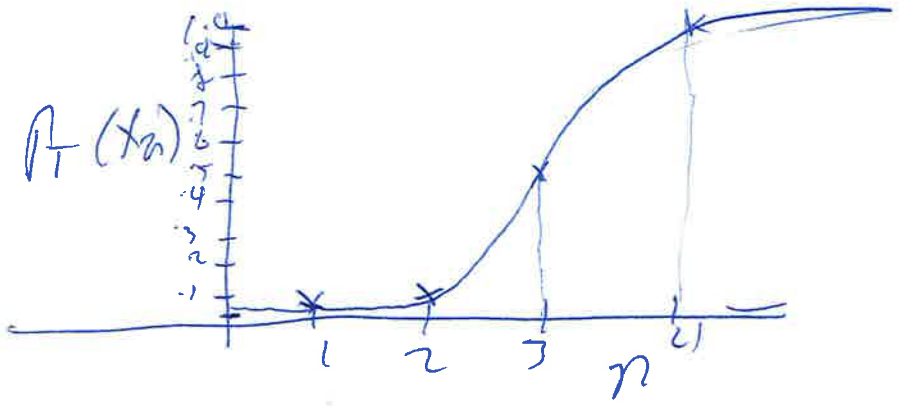
and $\varepsilon = .1$ then $P_T(X_n) = \frac{1}{1 + 1/10^{n-1}} = \frac{10^{n-1}}{1+10^{n-1}}$

$P_T(X_1) = \quad x = .01$

$P_T(X_2) = \dfrac{1}{1-.1+10} \simeq .1$

$P_T(X_3) = \dfrac{1}{1-.01+1} \simeq 1/2$

$P_T(X_4) = \dfrac{1}{1-.001+.1} \simeq 1$



Notice if in $r(X_2 X_1) = \frac{1}{x(1-\varepsilon)+\varepsilon}$

we put $\varepsilon = 1$ then $r = 1$

so if we falsified with respect to X_2 as well as X_1 when we factored for X_1 then no extra build of P_T

i.e. if theories are robust w.r.t. X_2 also, ~~they are~~ then X_2 gives no improvement in P_T.

So P_T increases if the ~~q~~ X_2 q is non-robust w.r.t. to ~~q~~ X_2 so ε is small.

P_T remains unchanged if ~~X is~~ q is robust w.r.t. X_2 so $\varepsilon = 1$.

now take X_2 ~~falsified~~ is false

then $P(X_2 | T) = 0$

and $P(T | X_1 X_2) = 0$

so in this case of theory makes a false prediction $P_T(X_2, X_1) \to 0$.

But to get back to verisimilitude we want to explain how a false theory can still be highly regarded.

So Take: $P(q \text{ is } T / X)$

$\to P_T(q / X)$

$\to P_T(C \,\&\, B_1 / X)$

$\to P(C \,\&\, B / X)$

C = hard core
$B_1 \ldots B_n$ = protective belt [illegible] [illegible] of the heuristic.

Then $P(C \,\&\, B_1 / X) = 0$ if X is false.

But $P(C \,\&\, B_1 / X) = P(B_1 / C \,\&\, X) \times P(C/X)$

$= 0 \Rightarrow$ either $P(C/X) = 0$

or $P(B_1 / C \,\&\, X)$

Now we assume $P(B_1 / C \,\&\, X) / P(C/X) = $ const.

So as long as X is [illegible].

Then $P(B / C \,\&\, X) = \sqrt{\Omega} \times P(B/C)$

$P(C/X) = \sqrt{\Omega}\; P(C)$

Now ~~take~~ if X is false
take $P(B_1|C,X) = 0$
$P(C|X)$ unchanged

Now continue to build up $P(C|X)$
with a new B_2 that explains X
until this is falsified.

then finally if no B_n, $i = 1 .. n$
is able to explain X, we can
build $P(C|X) = P(C,B_1|X)$
$+ P(C,B_1|X)$
$+ \ldots$

So $P(C|X)$ must
now be zero. Each term is zero

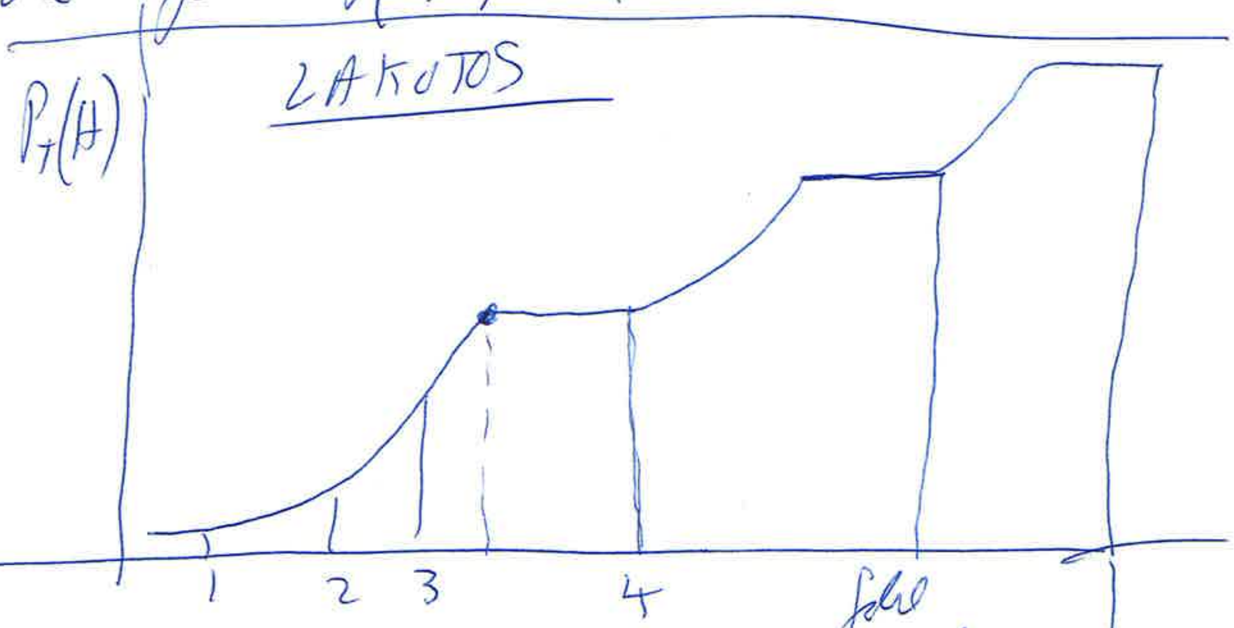
Once we have observed (C,B_1) or (C,B_2) or —

we $P(C|X) \underbrace{\sum_n P(B_n|C,X)}_{=1} = \overset{0}{\sum_n P(C,B_n|X)}$

Thus we are forced to take $P(C|X) =$

So picture for $P(H/X_n)$ under

$\cancel{P(H/X_n)} \neq$
$= 1-\varepsilon$

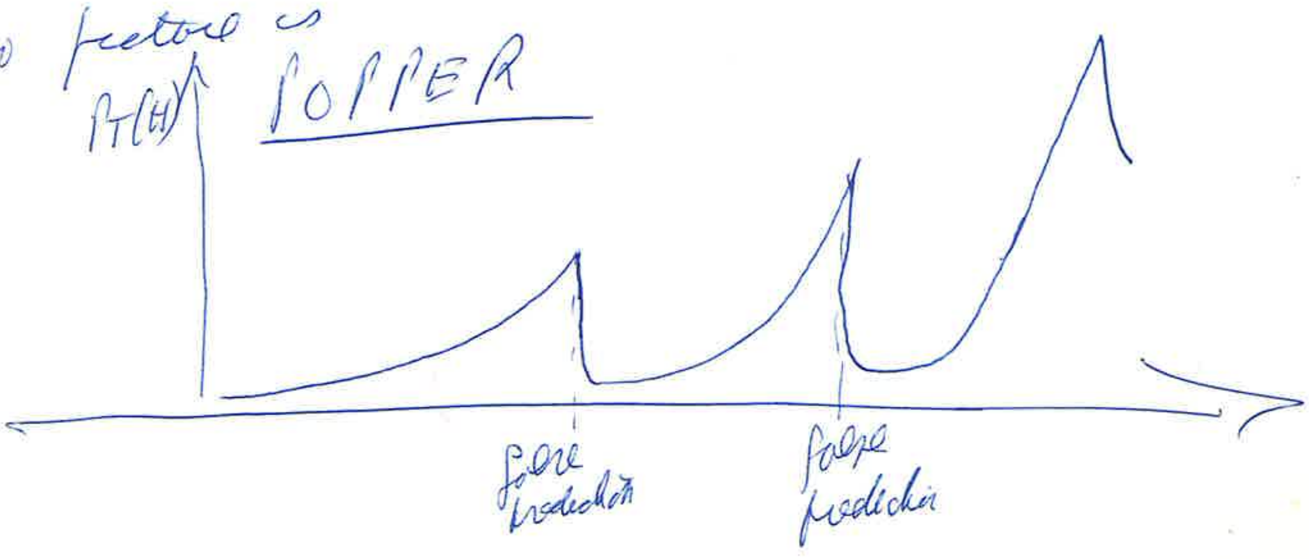


$$P(H/X_n) = \left(1-\varepsilon^n + \varepsilon^n/x\right)^{-1/2}.$$

ε = Prob (prediction is true / theory is false)
x = Prob (initial theory is true)
i.e. before any prediction

Popper's methodology if X is false then $P(H/X) = 0$

so picture is



we are given $P(B \supset C) = 0 = P(B|C)P(C)$

$\Longrightarrow$ $P(B|C) = 0$

or $P(C) = 0$ and $P(B|C)$ is undefined

we suppose $P(B|C)$ is independent of C's

so $P(B_1 \supset C) = P(B_1) \times P(C)$

and for Bayesian take methodological rule. $P(B_1) = 0$

and $P(C)$ is left alone.

For new hypothesis B_2 take $P(C)$ as before

and $P(B \supset C) = \underset{\substack{\text{old value} \\ \text{unchanged} \\ = \sqrt{E}}}{P(B_2)} \times \underset{\substack{\uparrow \\ \text{improved value.}}}{P(C)}$

now how do we decide to apportion

or + $P(B \not\supset C)$ between $P(B)$, $P(C)$

then $P(B) \mp P(C)$.

we want do this in a way that is

simple, accords with constraints of

the probability calculus.

Problem is this: write $P(A) \to A$ as

for $rAB < 1$ so max value of r is $1/AB$

and write $rAB = (\sqrt{r} \cdot \underset{T_1}{h(r)} \cdot A)(\sqrt{r}\, \underset{T_2}{B/h(r)})$

then choose $h(r)$ to satisfy

$$\begin{cases} h(r) = 1 & \text{for } r = 1 \\ h(r) = \sqrt{B/A} & \text{for } r = 1/AB \end{cases}$$

then $T_1 \to \frac{1}{\sqrt{AB}} \cdot \sqrt{B/A} \cdot A$

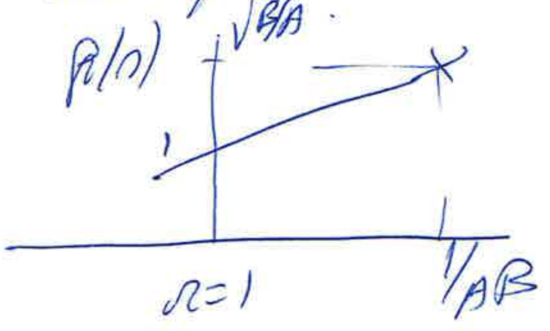
$\implies 1$ as r reaches its maximum

$\sqrt{r}\, h(r)$ increases in $(1, \frac{1}{AB})$

Also $\sqrt{r}/h(r)$ increases in $(1, \frac{1}{AB})$?

$T_2 \to \frac{1}{\sqrt{AB}} \cdot B \cdot \sqrt{\frac{A}{B}} \to 1.$

Simplest assumption is to determine $h(r)$ by linear interpolation



Then $h(r) = 1 + \dfrac{\sqrt{B/A} - 1}{\frac{1}{AB} - 1}(r - 1)$

So writing $\Lambda AB = (\Lambda_A \cdot A) \times \Lambda_2(B)$

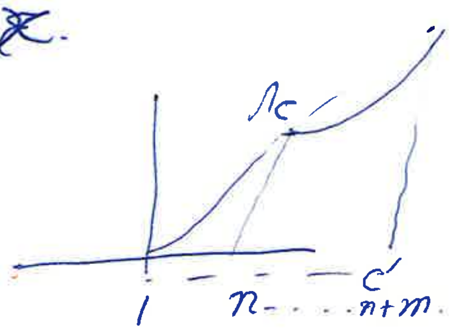
we have.

$$\Lambda_A = \sqrt{\Lambda}\left[1 + \frac{\sqrt{B/A} - 1}{\frac{1}{AB} - 1}(\Lambda - 1)\right]$$

~~where~~. So in the case of a load case C confined to a factor Λ_C

"Initial" values of ρ_e is $\sqrt{\Lambda_C}\sqrt{\bar{x}}$
— ρ_B . . $\sqrt{\bar{x}}$.

and later values for ρ_e



is $\rho_C = \sqrt{\Lambda_e^{(1-n)}\sqrt{\bar{x}}} \times \sqrt{\Lambda_e^{(n-m)}}$

~~$\left[1 + \left(\Lambda_C^{1-n}\right)^{-1/4}\right.$~~

$$\left[1 - \frac{1 - \left(\Lambda_e^{(1-n)}\right)^{-1/4}}{\left(\bar{x}\sqrt{\Lambda_e^{(1-n)}}\right)^{-1} - 1}\left(\Lambda_e^{(n+m)} - 1\right)\right]$$

where $\Lambda^{n+m} = \left(1 - \varepsilon^m + \dfrac{\varepsilon^m}{\sqrt{\Lambda^{(1,n)}}\,\bar{x}}\right)^{Ab/2\,-1} \Big/ \sqrt{\Lambda^{(1,n)}} \cdot x$

and $\Lambda^{(1,n)} = \left(1 - \varepsilon^n + \dfrac{\varepsilon^n}{x}\right)^{Ab/2\,-1} \Big/ x$

So n predictions successful then failure then m further predictions

$$P_c^{(n+m)} = \sqrt{\Omega^{(1,n)} \cdot \Omega^{(n,m)} \cdot x}$$

$$\times \left[1 - \frac{1 - \left(\Omega^{(1,n)}\right)^{-1/4}}{\left(x\sqrt{\Omega^{(1,n)}}\right)^{-1} - 1} \left(\Omega^{(n,m)} - 1\right) \right]$$

where $\Omega^{(n,m)} = \left(1 - \varepsilon^m + \dfrac{\varepsilon^m}{\sqrt{\Omega^{(1,n)} \cdot x}}\right)^{-1/2} \Big/ \sqrt{\Omega^{(1,n)} \cdot x}.$

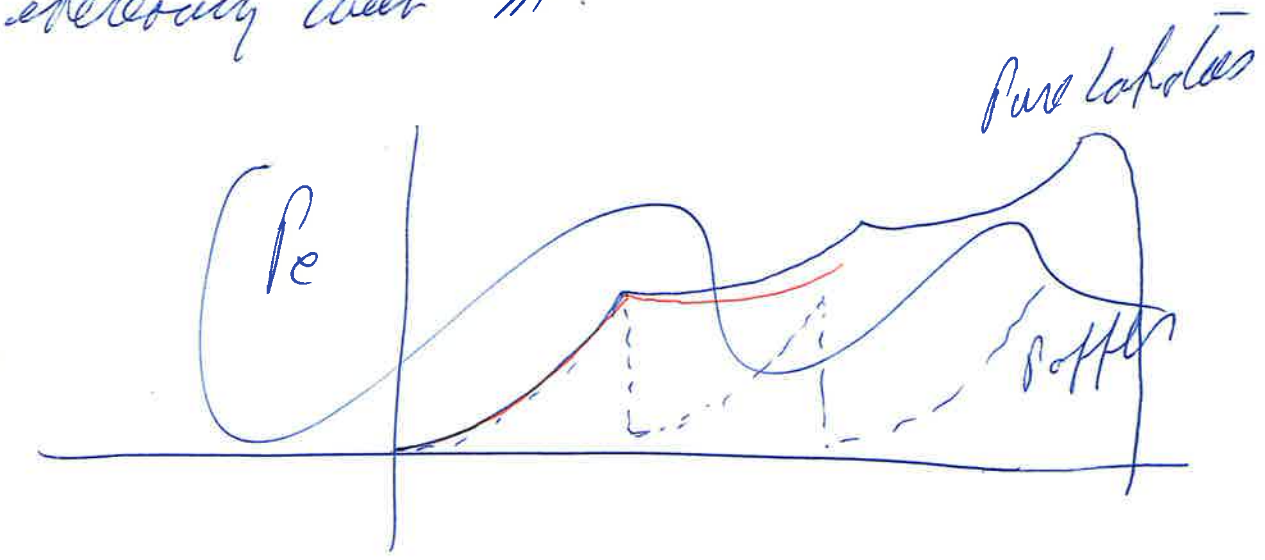
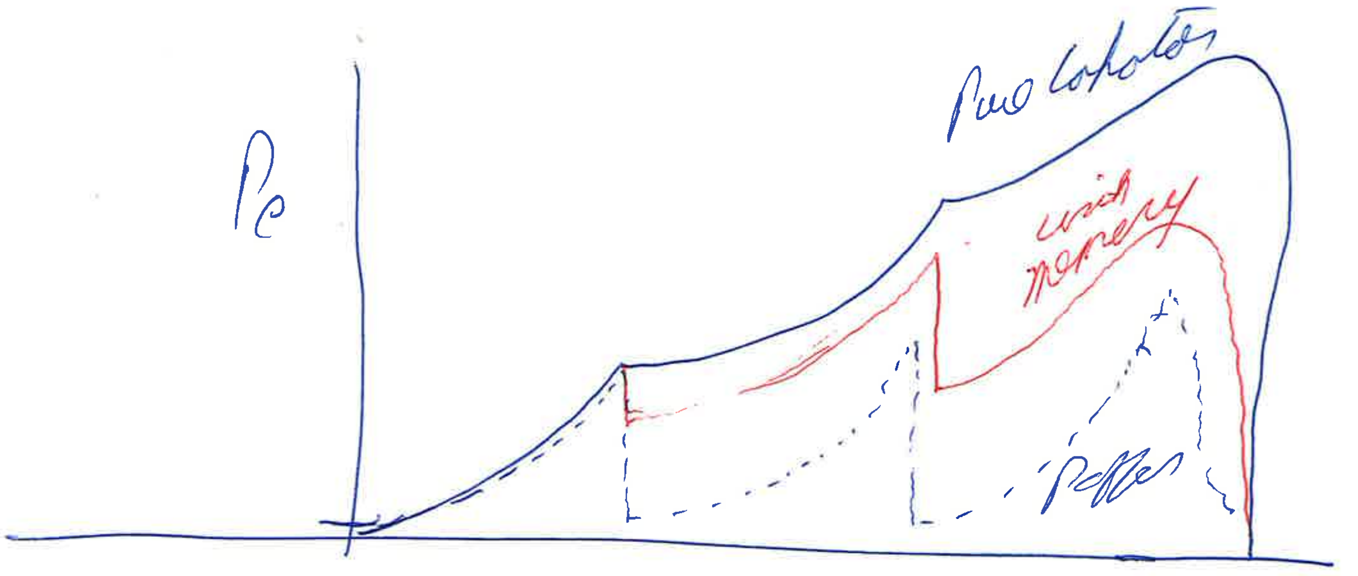
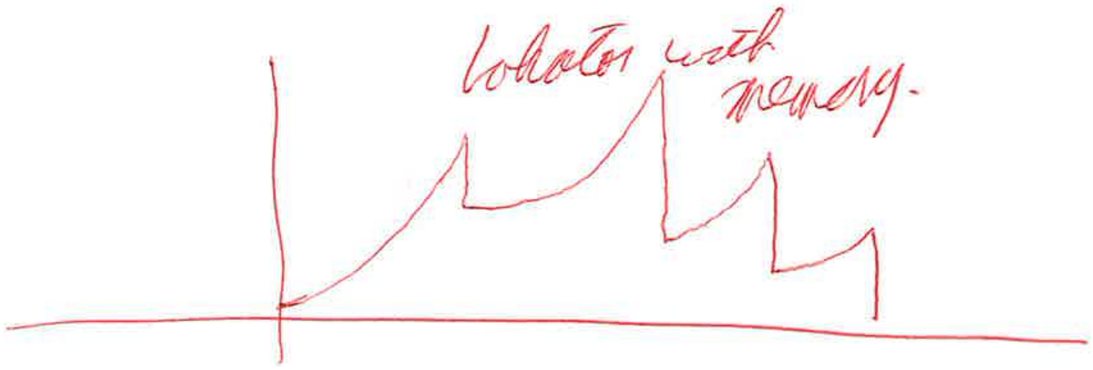
and $\Omega^{(1,n)} = \left(1 - \varepsilon^n + \dfrac{\varepsilon^n}{x}\right)^{-1/2} \Big/ x.$

ε $\cancel{\varepsilon}$ = prob (true prediction / theory is false)

x $\cancel{\varepsilon}$ = prob (theory is true / background knowledge)

So $\varepsilon = P(X \mid \neg T)$

$x = P(T / B)$ $\}$

We can now build in memory factors. after m duplication's. we drop $P_c^{(m)}$ by a factor increasing with m.